Series P/PB2/1

SET 1

CLASS XII PRE-BOARD EXAMINATION – 2024-25

Q.P. Code: 041/2/1

Roll No. | | | | | | | | |

Candidate must write the Q.P. Code on the title page of the answer-book.

- Please check that this question paper contains **8** printed pages.
- Please check that this question paper contains **38** questions.
- Q.P. Code given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.
- **Please write down the serial number of the question in the answer-book before attempting it.**
- 15 minute time has been allotted to read this question paper. The students will read the question paper only and will not write any answer on the answer-book during this period.

MATHEMATICS

Time allowed : 3 hours *Maximum Marks : 80*

General Instructions:

Read the following instructions very carefully and strictly follow them :

(i) *This question paper contains **38** questions. All questions are **compulsory**.*

(ii) *This question paper is divided into five Sections – **A, B, C, D** and **E.***

(iii) *In **Section A**, Questions no. **1 to 18** are multiple choice questions (MCQs) and questions number **19** and **20** are Assertion-Reason based questions of **1** mark each.*

(iv) *In **Section B**, Questions no. **21 to 25** are very short answer (VSA) type questions, carrying **2** marks each.*

(v) *In **Section C**, Questions no. **26 to 31** are short answer (SA) type questions, carrying **3** marks each.*

(vi) *In **Section D**, Questions no. **32 to 35** are long answer (LA) type questions carrying **5** marks each.*

(vii) *In **Section E**, Questions no. **36 to 38** are case study based questions carrying **4** marks each.*

(viii) *There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and 2 questions in Section E.*

(ix) *Use of calculators is **not** allowed.*

SECTION A

Directions(Q.Nos. 1 to 20): Multiple Choice Questions).Each question carries 1 mark.

1 The domain of $\sec c^{-1}(3x-2)$ is

(a) $\left(-\infty, \frac{1}{3}\right]$ (b) $[1, \infty)$ (c) $(-\infty, -1]\cup[1, \infty)$ (d) $\left(-\infty, \frac{1}{3}\right]\cup[1, \infty)$

2 The maximum value of $\sin x \cos x$ is

(a) $\frac{1}{2}$ (b) $\frac{1}{4}$ (c) $\sqrt{2}$ (d) $2\sqrt{2}$

3 If A is a square matrix satisfying $A^2 = A$, then $(A – I)^3 + (A + I)^3 – 7A$ is equal to

(a) A (b) I – A (c) I + A (d) 3A

4 If $A.(adjA)=\begin{bmatrix}-3 & 0 & 0\\ 0 & -3 & 0\\ 0 & 0 & -3\end{bmatrix}$, then the value of $|A|+|adj\ A|$ is equal to

(a) 9 (b) 12 (c) 6 (d) 27

5 If $f(x) = \begin{cases} mx+1 & if\ x\le\frac{\pi}{2} \\ \sin x+n & if\ x>\frac{\pi}{2}\end{cases}$ is continuous at $x=\frac{\pi}{2}$, then

(a) $m=1, n=0$ (b) $m=\frac{n\pi}{2}+1$ (c) $n=\frac{m\pi}{2}$ (d) $m=n=\frac{\pi}{2}$

6 $\int_0^{2/3}\frac{1}{4+9x^2}\,dx$ equals

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{12}$ (c) $\frac{\pi}{24}$ (d) $\frac{\pi}{4}$

7 For what value of $x\in\left[0, \frac{\pi}{2}\right]$, *is* $A+A^T=\sqrt{3}I$, *where* $A=\begin{bmatrix}\cos x & \sin x\\ -\sin x & \cos x\end{bmatrix}$?

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{6}$ (c) $\frac{\pi}{2}$ (d) 0

8 $\int\frac{e^x(1-\sin x)}{1-\cos x}\,dx$ equals

(a) $-e^x\tan\frac{x}{2}+C$ (b) $-e^x\cot\frac{x}{2}+C$ (c) $e^x\cot\frac{x}{2}+C$ (d) $-2e^x\cot\frac{x}{2}+C$

9 The sum of the order and degree of the differential equation $\frac{d}{dx}\left(\left(\frac{dy}{dx}\right)^3\right)=0$ is

(a) 2 (b) 5 (c) 3 (d) 0

10 The objective function $Z = ax + by$ of an LPP has maximum value 62 at (4 , 6) and minimum value 29 at (3 , 2). Then the value of $a \ and \ b$ is

(a) $a = 5, b = 7$ (b) $a = 7, b = 5$ (c) $a = 5, b = 2$ (d) $a = 3, b = 5$

11 $\int_0^{\pi/2} \frac{\sqrt[3]{\sin x}}{\sqrt[3]{\sin x} + \sqrt[3]{\cos x}} dx = a \frac{\pi}{2}$, then the value of a is

(a) 1 (b) $\frac{1}{2}$ (c) 2 (d) 4

12 If $\vec{a}$ and $\vec{b}$ are two unit vectors inclined to X-axis at angles 30^0 and 120^0 respectively, then $\left|\vec{a} + \vec{b}\right|$ equals

(a) 2 (b) $\sqrt{\frac{2}{3}}$ (c) $\sqrt{3}$ (d) $\sqrt{2}$

13 Let A be a skew-symmetric matrix of order 3. If $|A| = x$, then $(2025)^x$ is equal to

(a) 2025 (b) $(2025)^2$ (c) $\frac{1}{2025}$ (d) 1

14 If $\begin{bmatrix} x & 2 \\ 3 & x-1 \end{bmatrix}$ is a singular matrix, then the product of all possible values of x is

(a) 6 (b) -6 (c) 5 (d) 0

15 The number of arbitrary constants in the particular solution of a differential equation of third order is

(a) 0 (b) 2 (c) 1 (d) 3

16 The number of feasible solutions of the linear programming problem given as

Maximize $Z = 15x + 30y$

Subject to : $3x + y \le 12$

$x + 2y \le 10$, $x \ge 0,\ y \ge 0$ is

(a) 4 (b) 2 (c) 3 (d) Infinite

17 If $\left|\vec{a}\right| = 3$ and $-1 \le k \le 2$, then $\left|k\vec{a}\right|$ lies in the interval

(a) $[3, 6]$ (b) $[-3, 6]$ (c) $[1, 2]$ (d) $[0, 6]$

18 The probability that A speaks truth is $\frac{4}{5}$ and that of B speaking the truth is $\frac{3}{4}$. The probability that they contradict each other in stating the same fact is

(a) $\frac{7}{20}$ (b) $\frac{1}{5}$ (c) $\frac{3}{20}$ (d) $\frac{4}{5}$

Directions (Q.Nos. 19 to 20): In the following questions, a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct option:

a. **Both Assertion(A) and Reason (R) are true and Reason(R) is the correct explanation of assertion(A).**
b. **Both Assertion(A) and Reason (R) are true but Reason(R) is not the correct explanation of assertion(A).**
c. **Assertion(A) is true but Reason (R) is false.**
d. **Assertion (A) is false but Reason (R) is true.**

19 Assertion (A) : A line through the points A (4 , 7 , 8) and B (2, 3, 4) is parallel to a line through the points P(– 1 , – 2 , 1) and Q(1 , 2 , 5).

Reason(R): Lines $\vec{r} = \vec{a}_1 + \lambda \vec{b}_1$ and $\vec{r} = \vec{a}_2 + \mu \vec{b}_2$ are parallel if $\vec{b}_1 . \vec{b}_2 = 0$

20 Assertion(A): If $f(x) = |\cos x|$, then the derivative of $f(x)$ *at* $x = \dfrac{3\pi}{4}$ *is* $\dfrac{1}{\sqrt{2}}$

Reason(R) : $f(x) = |\cos x| = \begin{cases} \cos x & if\ 0 \le x \le \dfrac{\pi}{2} \\ -\cos x & if\ \dfrac{\pi}{2} < x \le \pi \end{cases}$

SECTION – B

Directions(Q.Nos.21 to 25): This section comprises of Very short answer type questions (VSA) of 2 marks each.

21 If $y = \sqrt{\dfrac{\sec x - 1}{\sec x + 1}}$, find $\dfrac{dy}{dx}$ *at* $x = \dfrac{\pi}{3}$

OR

If $\sin y = x \cos(a + y)$, then show that $\dfrac{dy}{dx} = \dfrac{\cos^2(a + y)}{\cos a}$

22 The Cartesian equation of a line are $6x - 2 = 3y + 1 = 2z - 2$. Find its direction ratios and also write the vector equation of the line.

23 Find the value of $\sin^{-1}\left[\cos\left(\dfrac{33\pi}{5}\right)\right]$.

24 The radius r of a right circular cone is decreasing at the rate of 3 cm/minute and the height h is increasing at the rate of 2 cm/minute . When r = 9 cm and h = 6 cm, find the rate of change of its volume.

25 Evaluate: $\displaystyle\int \frac{\sin(x - a)}{\sin(x + a)}\, dx$

OR

Evaluate: $\int \frac{dx}{\sqrt{5-4x-2x^2}}$

SECTION – C

Directions(Q.Nos.26 to 31): This section comprises of short answer type questions (SA) of 3 marks each.

26 Find the general solution of the following differential equation:

$$x\frac{dy}{dx} + y - x + xy\cot x = 0\ (x \neq 0).$$

27 Find the value of p *and* q for which $f(x) = \begin{cases} \frac{1-\sin^3 x}{3\cos^2 x} & if \quad x < \frac{\pi}{2} \\ p & if \quad x = \frac{\pi}{2} \\ \frac{q(1-\sin x)}{(\pi - 2x)^2} & if \quad x > \frac{\pi}{2} \end{cases}$ is continuous at $x = \frac{\pi}{2}$.

OR

If $x = \cos t + \log\left(\tan\frac{t}{2}\right)$, $y = \sin t$, then find the value of $\frac{d^2y}{dx^2}$ *at* $t = \frac{\pi}{4}$.

28 A bag contains $(2n+1)$ coins. It is known that $(n-1)$ of these coins have a head on both sides whereas the rest of the coins are fair. A coin is picked up at random from the bag and is tossed. If the probability that the toss results in a head is $\frac{31}{42}$, determine the value of n.

OR

A committee of 4 students is selected at random from a group consisting of 6 boys and 4 girls. Given that there is at least one girl in the committee, calculate the probability that there are exactly two girls in the committee.

29 Solve the following linear programming problem graphically.

Maximize $Z = 105x + 90y$

Subject to the constraints:

$x + y \leq 50$

$2x + y \leq 80$

$x \geq 0,\ y \geq 0$

30 Evaluate $\int \frac{x^2}{x^4 + x^2 - 2}\,dx$

OR

Evaluate $\int_0^{\pi/4} \left(\sqrt{\tan x} + \sqrt{\cot x}\right) dx$

31 Find t*h*e intervals in which the function $f(x) = x^4 - \frac{x^3}{3}$ is (a) strictly increasing (b) strictly decreasing.

SECTION – D

Directions(Q.Nos.32 to 35): This section comprises of Long answer type questions (LA) of 5 marks each.

32 Show that a function $f : R \to R$ defined by $f(x) = \frac{2x}{1+x^2}$ is neither one-one nor onto. Further, find set A so that the given function $f : R \to A$ becomes an onto function.

33 If A_1 denotes the area of region bounded by $x^2 = 8y$, $y = 1$ and Y-axis in the first quadrant and A_2 denotes the area of region bounded by $x^2 = 8y$, $y = 4$, find $A_1 : A_2$.

34 To control a crop disease it is necessary to use 8 units of chemical A, 14 units of chemical B and 13 units of chemical C. One barrel of spray P contains one unit of A, 2 units of B and 3 units of C. One barrel of spray Q contains 2 units of A, 3 units of B and 2 units of C. One barrel of spray R contains 1 unit of A, 2 units of B and 2 units of C. If spray P, Q and R cost ₹500, ₹250 and ₹200 per barrel respectively, find the total cost incurred using matrices.

OR

If A, B and C are angles of a triangle, then prove that $\begin{vmatrix} -1 & \cos C & \cos B \\ \cos C & -1 & \cos A \\ \cos B & \cos A & -1 \end{vmatrix} = 0$

35 Find the vector and Cartesian equations of a line passing through the point (1 , 2 , – 4) and parallel to the line joining the points A(3, 3, – 5) and B(1 , 0 , – 11). Hence find the distance between the two lines.

OR

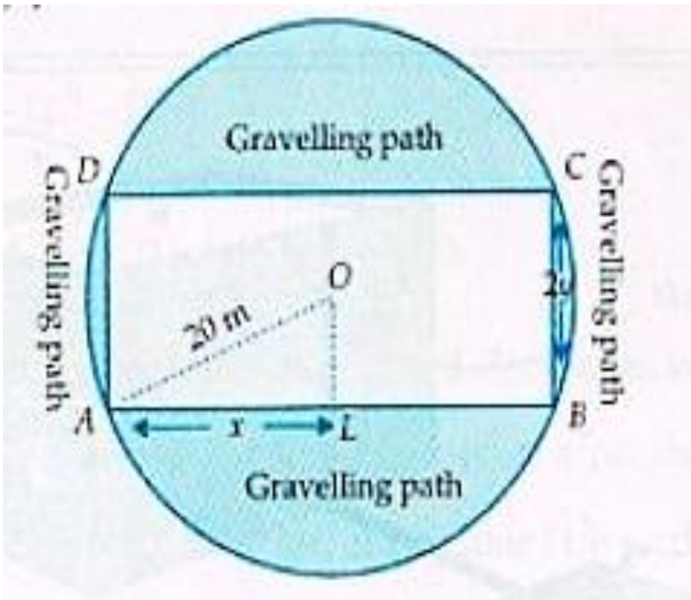
Find the value of b so that the lines $\frac{x-1}{2} = \frac{y-b}{3} = \frac{z-3}{4}$ and $\frac{x-4}{5} = \frac{y-1}{2} = z$ are intersecting lines. Also, find the point of intersection of these given lines.

SECTION – E

This section comprises of 3 case-study/passage-based questions of 4 marks each with sub-parts.

36 **Case-Study 1:**

An architect designs a garden in a residential complex. The garden is in the shape of a rectangle inscribed in a large circle of radius 20 m as shown in the figure. Let $2x$ *and* $2y$ be the length and breadth of the rectangular garden.



Based on the above information answer the following questions:

(i) Find the area A of the garden in terms of x. (1)

(ii) Find $\frac{dA}{dx}$ (1)

(iii) Find the maximum area of the garden. (2)

OR

(iii) Find the area of gravelling path. (2)

37 **Case Study 2:**

At the start of a cricket match, a coin is tossed and the team winning the toss has the opportunity to choose to bat or bowl. Such a coin is unbiased with equal probabilities of getting head and tail.

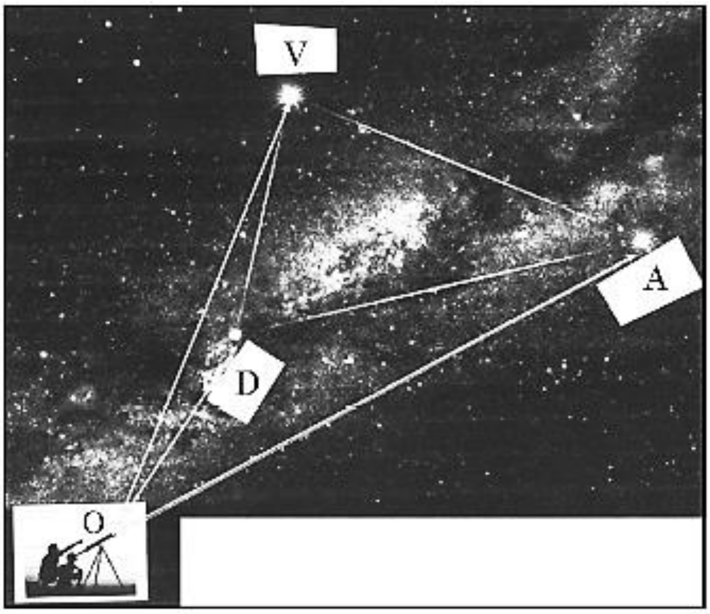
Based on the above information, answer the following questions:

(i) If such a coin is tossed two times, then find the probability distribution of number of tails. (2)

(ii) Find the probability of getting at least one head in three tosses of such a coin. (2)

38 **Case Study 3:**

An instructor at the astronomical centre shows three among the brightest stars in a particular constellation. Assume that the telescope is located at O(0 , 0 , 0) and the three stars have their locations at the points D, A and V having position vectors $2\hat{i}+3\hat{j}+4\hat{k}$, $7\hat{i}+5\hat{j}+8\hat{k}$ and $-3\hat{i}+7\hat{j}+11\hat{k}$ respectively.



Based on the above information, answer the following questions:

(i) How far is the star V from star A? (1)

(ii) Find a unit vector in the direction of $\overrightarrow{DA}$. (1)

(iii) Find the measure of $\angle VDA$. (2)

OR

(iii) Find the projection of vector $\overrightarrow{DV}$ on vector $\overrightarrow{DA}$? (2)